# Numerical Modeling of the Order of Convergence of Surface Charge Density Useful in Estimating Bandwidth for Microstrip Patch Antenna

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***Abstract*—In spite of many advantages, microstrip patch are affected by the disadvantage of low having low bandwidth. In addition to many other factors, that determine the bandwidth of these antennas, one important parameter is the shape of the patch.The idea suggested in this paper is relating the proper shape of microstrip patch antenna for higher bandwidth operations using the phenomenon of fringing on finite surface of patch and estimating its occurrence from numerical integration.For this purpose, surface charge density on metallic surfaces is taken under consideration. This is obtained by carrying out computations on a geometric sequence of cell sizes. The results are then employed on microstrip patch antenna to make conclusions about their bandwidth of operation.**

***Keywords*—Microstrip patch antenna, bandwidth, shape, fringing, numerical integration, surface charge density, cell size.**

## I. INTRODUCTION

THE sources of potentials and fields, are surface charges. In a physical system the arrangement of surface charges are used to reproduce the applied potentials. If all the surface charges are known then by using Coulomb's Law we can calculate the potentials and fields in any part of the system vicinity.

Equally important is, the determination of electric and magnetic fields in the far field regions of an antenna. These require firstly the determination of electric and magnetic sources which in addition to vector electric and magnetic current densities, also include surface charge densities, as defined by Gauss's law in Maxwell's equations [1].

Charge distributions on the metal surface determine the current densities which establish the radiation characteristics of the radiating antenna.Therefore, very importantly, parts of the system that a designer has to model in finding antenna radiation are the surfaces of the metals which radiate electromagnetic fields.

For numerical computation of surface charge density the methodology to proceed is by subdividing the metal surfaces into segments, which are either triangles or rectangles (depending on the shape of the antenna radiating surface) and then usestandard computational electromagnetic tool/ algorithm, such as FEM technique or MoM to analytically find radiation behavior.

In this paper we have used numerical integration method to determine surface charge densities using their modeled equation. The antenna taken under consideration is a rectangular microstrip patch antenna.

The focus is, in finding and estimating the order of convergence by carrying out computations on a geometric sequence of cell sizes. This gives accurate results for regular integrands. However for singular integrals we need to estimate the order of convergence. Singular integrals occur on the sides, corners or edges of the surfaces (under consideration). This paper elaborates the usage of a convenient way to simulate these estimations, as explained in [2]. The estimated order of convergence is then used to find the model value of the integrand from extrapolation to zero mesh cell size. This model value is sum of the desired solution and error. Extrapolated exact solution can further be improved by increasing the rate of convergence. This is normally accomplished by sequence acceleration method, called Richardson extrapolation [4-5].

## II. PROBLEM DEFINITION

Considered a microstrip patch antenna connected to an electromagnetic source. When the patch is energized, both the upper and lower patch surfaces and the ground plane will establish a charge distribution. This charge distribution is shown in the following Fig. 1.In the figure, $J_b$ and $J_t$ are bottom and top surface currents.Charge distribution is controlled by two mechanisms; the force of attraction and the force of repulsion. Attractive mechanism is between the bottom side of the patch and theground plane, which tends to maintain the charge concentration on the bottom of thepatch. The repulsive mechanism is between like charges on the bottom surface of thepatch, which tends to push some charges from the bottom of the patch, around itsedges, to its top surface. The movement of these charges creates corresponding current densities J band $J_t$, at the bottom and top surfaces of the patch, respectively.

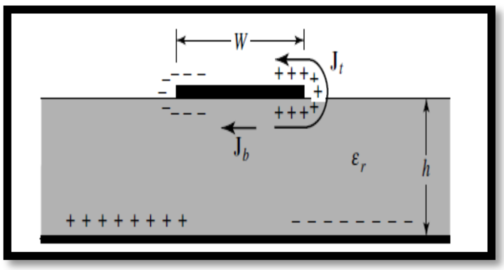


Fig. 1.Charge distribution and current density creation on microstrip patch [7].

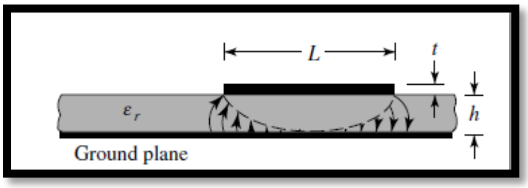


Fig. 2. Electric Field Lines on microstrip patch [7].

Thus electric fields exist between the metal patch and metallic ground plane just like in a two plate capacitor as shown in the Fig. 2.To be noted over here, is the fringing of the fields at the ends/ corners of the patch.These fringing fields determine the broadside radiation pattern of the antenna as explained in [6].

Surface charge density becomes singular on sharp metal edges and corners.

This becomes problematic, when total charge needs to be found out. Surface charge density close to the edges varies as power of the distance between the plates to the angle subtended by the metal edge. This leads to integration which can be either of regular type or singular. Regular integrands give the general behavior of charge distribution on the surface. However if the power of distribution is less than zero and/or lower limits of integration is equal to zero, then we have a singular integrand. This has physical significance of the surface charge distribution in the vicinity of a sharp metal edge, where the fringing phenomenon occurs. Mathematically if "$x$" is the distance to the edgeof the patch and taking the patch and ground plane as two metal plates then surface charge density varies as $x^{-(\pi-\beta)/(2\pi-\beta)}$ [2] , here β is the angle subtended by the edge. This angle, which is also taken as power of $x$, is obtained by subtracting from 180o and then normalizing it by subtracting the angle from 360o.To simplify this scenario we can approximate following integration equation for the above mentioned problem as

$$\int_a^b x^{\xi}\, dx \qquad \textbf{(1)}$$

Here, $\xi = -(\pi-\beta)/(2\pi-\beta)$

*Conditions:*

*Case 1:*

$a = 0$ together with power of $x < 0$ gives singular integrand.

*Case 2:*

$a > 0$ together with power of $x > 0$ gives regular integrand or regural surface.

It is obvious that for β=0 (indicating that the plates are very thin), we have ξ=-1/2 and this indicates smallest possible value of ξ in the given electrostatic problem.

In this paper surface charge distribution near the corners and edges is modeled numerically, and then estimation of the order of convergence for singular case is determined. Lower order of convergence indicates that we are at the edges or corners and this is where the fringing of the fields occurs. So by determining the order of convergence, we obtain the fringing positions. The more we have these fringing of the fields the more bandwidth of patch antennas can be obtained.The methodology described in this paper includes numerically finding the surface charge density, then estimating the order of convergence, to obtain positions of fringing. This work can be extended to obtain new shapes for patch antennas which give more bandwidth.

## III. METHODOLOGY

Authors have worked on statistical modeling of patch antennas [8]. The methodology being discussed here is taken from [2-3] where surface charge density for capacitor plates was numerically computed. In this paper, we have used the same idea on microstrip patch antenna surface.In Matlab,numerical method of midpoint integration was computed and simulated with N subintervals and the length of each interval is expressed as h, with h defined as,

$$h = (b-a)/N$$

In the numerical tests described below, the value of the numerically evaluated integral is denoted by 'Imodel(h)' and analytically evaluated integral is denoted by ' Io '. Difference between IoandImodel(h) is given by error, "e" in numerical integration. In the numerical tests that were performed we took hypothetical values of a = 1andb = 2. The values indicate linear dimensions of radiating surface of patch antenna. The values of N and corresponding h that were used and obtained respectively are as follows:

$$N = [10 \quad 20 \quad 40 \quad 80 \quad 160]$$
$$h = [\,0.2000 \quad 0.1000 \quad 0.0500 \quad 0.0250 \quad 0.0125]$$

Increasing the value of $N$, make the area under test higher in resolution and therefore lesser error.

Analytically order of convergence '$\alpha$', can be calculated from the following expression [2].

$$\alpha = \ln\left(\frac{I(h_i)-I(h_{i+1})}{I(h_{i+1})-I(h_{i+2})}\right)/\ln\left(\frac{I(h_i)}{I(h_{i+1})}\right) \qquad \textbf{(2)}$$

Solution for the Imodel(h) is sum of the desired solution and an error term which is a polynomial in terms of mesh size h.

$$Imodel(h) = c_o + c_\alpha h^\alpha \qquad \textbf{(3)}$$

The desired result is '$c_o$', '$c_\alpha$' and 'α' are all unknown. $c_o$ is the result extrapolated to zero mesh size. To fit the mesh size we need to know the order of convergence, i.e. 'α'. For estimation purpose a range of values of α can be tested against error.

To find the other two unknowns, i.e. $c_o$ and $c_\alpha$; either we can make use of the least square fit. The least square fit method utilizes twomatrices A and B to find the unknown matrix X by using the following equation

$$X = A\backslash B \qquad \textbf{(4)}$$

Where X'= [$c_o$ $c_\alpha$]

$$A=\begin{bmatrix}1 & h(1)^\alpha\\1 & h(2)^\alpha\\1 & h(3)^\alpha\\1 & h(4)^\alpha\\1 & h(5)^\alpha\end{bmatrix} B=\begin{bmatrix}Imidp(h(1))\\Imidp(h(2))\\Imidp(h(3))\\Imidp(h(4))\\Imidp(h(5))\end{bmatrix}$$

The power of x taken for integration is $-1/2, 1/2, 3/2$.

## IV. Observations and Results

Fig. 3 and 4 shows the estimated order of convergence obtained from numerical integration for the edges and regular surfaces respectively for Case 1 and Case 2 as stated previously.

## V. Conclusion and Comments

From the analysis made in this study we have shown the methodology for the correct estimation of the order of convergence from numerical computation of surface charge distribution on the metal surfaces which effect the radiation patterns of the patch antenna. We obtained low value of order of convergence (i.e. -1/2) for edge and a value equal to 2 for normal (not edged) surface. Thus by scanning the regions of surface with low values of order of convergence , edges could be detected and thus those regions could then be utilized for increasing bandwidth of operation of patch antennas giving it different shapes.

If has been researched that some of the specific patch shapes have more bandwidth then others. This increased in the bandwidth is because of the increase in the fringing area which in actuality is due to singularity and lower order of convergence at the edges.Table I shows a comparison of patch shapes with the bandwidth. It is to be noted that patch antennas with more strict edges have more bandwidth.

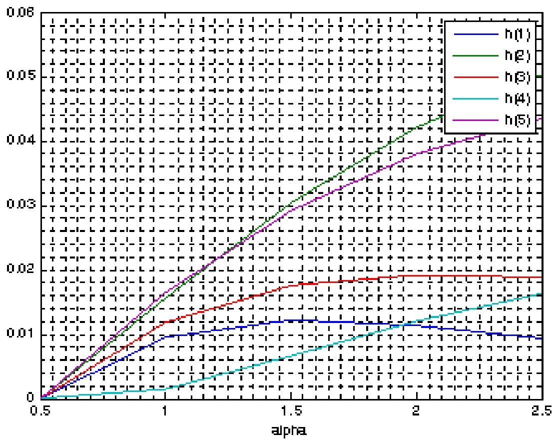


Fig. 3 .Simulations for Order of Convergence for range of α for different value of h when ξ=-1/2

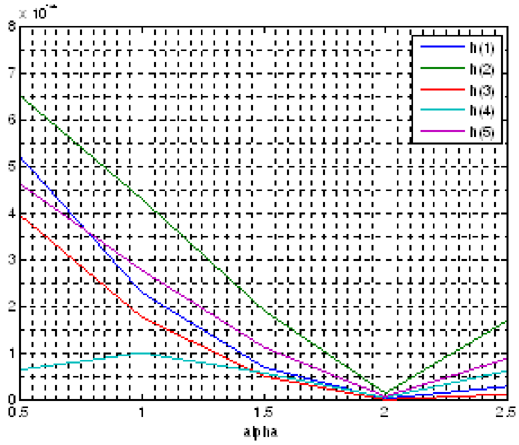


Fig. 4. Simulations for Order of Convergence for range of αFor different value of h when ξ=3/2

TABLE I: PATCH SHAPES AND VARYING BANDWIDTHS

| Patch Shape | Bandwidth (%) |
|---|---|
| Narrow rectangular patch | 0.7 |
| Wide rectangular patch | 1.6 |
| Square patch | 1.3 |
| Circular disk | 1.3 |
| Annular ring | 3.8 |

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