Performance Analysis of MMSE Channel Estimation in Multiuser Massive MIMO System under Log-Normal Shadowing

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***Abstract*—This paper analyzes the performance of MMSE channel estimation in multiuser massive MIMO system under the Log-Normal shadowing. In this system, there are K number of decentralized User Equipment (UE) with single antenna and large number of antennas at the base station (BS). Pilot symbols are utilized for time division duplex (TDD) channel estimation. Performance of MMSE channel estimation of two existing algorithms is analyzed at different values of power path-loss, at different number of users, and at different number of antennas at the BS in the system. Results show that the performance of channel estimation degrades with the increase in the number of antennas at BS and the number of users in the system due to increase in the channel path. Performance of the two algorithms in channel estimation is also compared for different values of power path loss.**

***Index Terms*—Multi-User Massive MIMO, Channel Estimation, MMSE, Downlink communications, Complex Gaussian channel, Pilot contamination, Log-Normal shadowing, Fast fading**

I. INTRODUCTION

In recent wireless communications, Massive MIMO system has immerged as a vital promise for future [1-3]. In the 5th generation wireless systems (5G), massive MIMO is a bright candidate among its counterparts to provide high speed and link reliability and hence has gained a lot of attention by the research community [1-7]. According to [3], massive MIMO system is a type of wireless communication in which the base station (BS) serves with very large number of antennas (in hundreds or more) in contrast to the number of users in the system. In another definition, any configuration in which more than the highest MIMO mode in LTE which is 8x8 is referred to as Massive MIMO system [5].

High promise by massive MIMO is limited by many factors such as pilot contamination, channel impairment, etc [8]. Thus, signal processing is another crucial aspect of massive MIMO system, which should advance significantly in 5G. The computational complexity of the signal processing at receiving and transmitting ends increases rapidly due to increase in the number of antennas. Thus, less complex solutions are required for processing tasks [8, 9].

The massive MIMO systems performance depends mainly on the propagation environment and the antenna properties [10]. The system performance will decrease due to imperfect channel state information (CSI) with which channel is estimated [11, 12]. There are many recent works that investigate the impact of pilot contamination and channel estimation in Massive MIMO system [13-16].

The communication scenario can be either uplink or downlink. In the work provided in [9], uplink scenario is considered. This work proposes that user equipment will have uncorrelated channel in the presence of great number of antennas at the BS and channel gain of each user can be estimated from its 2nd order statistics. User equipment channel vector should be known or estimated properly at the BS to utilize benefits of massive MIMO system [9].

In this work, we modify the Massive MIMO model to incorporate the Log-Normal shadowing in the existing solutions of two channel estimation algorithms [6, 9]. Thus, we investigate the MSE performance of these algorithms for various scenarios of the Log-Normal shadowing. Moreover, we also analyze the effect of different system parameters such as antenna size and different number of users, etc., on the performance of these algorithms.

The rest of this paper is arranged as follows. In next section the system model is introduced that is used in the performance comparison of massive MIMO under Log-Normal shadowing. Section III summarizes the two existing channel estimation techniques for massive MIMO system. Our proposed model is presented in Section IV and its advantages are outlined in Section V. The results of the analyses are discussed in Section VI. The last section (Section VII) concludes this paper.

II. SYSTEM MODEL

Multiuser system with uplink communication scenario is considered [9] in which base station contains M antennas and there are K single antenna users such that $M>>$K, that means, Massive Multiuser MIMO system is under observation. At symbol time t, the base station will receive an Mx1 vector y_t which is given by

$$\boldsymbol{y_t} = \sum_{i=1}^{K} \boldsymbol{h}_i x_{it}^{*} + \boldsymbol{n_t} \quad (1)$$

where $h_i C^{Mx1}$ is the channel vector between BS and the ith user, x_{it} is the pilot of the ith user transmitted at symbol period t and $n_t \; \epsilon \; C^{Mx1}$ is the vector of noise at time t. Furthermore, the channel is modeled as h_km=(g_km) h _km in which fast fading part is h_{km} and distant dependent path loss is $\sqrt{g_{km}}$. Suppose that $\sqrt{g_{km}}$ = $\sqrt{g_k}$,$\forall m$ and $\sqrt{g_k}$, $\forall k$ are known at the base station.

If the N symbol periods are utilized, the received signal y_t can be grouped into the following matrix Y

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X}^H + \boldsymbol{N} \tag{2}$$

where $\boldsymbol{X} = [x_1, x_2, x_3, \dots, x_K]$, $\boldsymbol{N} = [n_1, n_2, n_3, \dots, n_K]$ and $\boldsymbol{H} = [h_1, h_2, h_3, \dots, h_K]$.

III. REVIEW OF CHANNEL ESTIMATION ALGORITHMS FOR MASSIVE MU-MIMO

Channel estimation is very important in wireless communication; if channel is not estimated properly performance of the system degrades badly. In [6], MMSE channel estimation and existing pilot assignment are discussed. We present here two well-known channel estimation algorithms from the literature [6, 9].

A. Algorithm 1 [6]

From [6], the pilot symbols of all users satisfy the property of orthogonality, i.e., $X^H X = I_k$. Thus, after post multiplication of the received signal in (2) by the pilot matrix X, noisy estimate of the channel matrix H is obtained as

$$\boldsymbol{Z} \triangleq \boldsymbol{Y}\boldsymbol{X} = \boldsymbol{H} + \boldsymbol{N}\boldsymbol{X} = \boldsymbol{H} + \widetilde{\boldsymbol{N}} \tag{3}$$

As a result, the kth column of the matrix Z (denoted by z_k) is given by

$$\boldsymbol{z}_k = \boldsymbol{h}_k + \widetilde{\boldsymbol{n}}_k, \quad k = 1, \dots, K \tag{4}$$

where n_k the kth column of N. Now, to find the estimate of the channel for user k, the vector z_k is multiplied by the receiver weight matrix W_k^H, that is,

$$\widehat{\boldsymbol{h}}_k = \boldsymbol{W}_k^H \boldsymbol{z}_k \tag{5}$$

The receiver matrix W_k^H can be designed using well-known minimum mean-square-error (MMSE) criterion. Thus, we define the MMSE objective function as

$$\xi_k = E\left[\|\widehat{\boldsymbol{h}}_k - \boldsymbol{h}_k\|^2\right] \tag{6}$$

which can be shown to give

$$\xi_k = tr\{g_k \boldsymbol{W}_k^H \boldsymbol{W}_k - g_k \boldsymbol{W}_k^H - g_k \boldsymbol{W}_k + g_k \boldsymbol{I}_M + \sigma^2 \boldsymbol{W}_k^H \boldsymbol{W}_k\} \tag{7}$$

where σ^2 is the variance of noise n_k. Now, the optimal W_k can be obtained by evaluating the derivative of the above cost function and letting it to zero, that is,

$$\frac{\partial \xi_k}{\partial \boldsymbol{W}_k} = 0 \tag{8}$$

which results in

$$\boldsymbol{W}_k = \frac{g_k}{g_k + \sigma^2} \boldsymbol{I} \tag{9}$$

Pseudo-code of Algorithm 1

1: Initialize weights $\boldsymbol{W}_k$
2: **Repeat**
3: Find $\boldsymbol{Z}$ using $\boldsymbol{Z} \triangleq \boldsymbol{Y}\boldsymbol{X} = \boldsymbol{H} + \boldsymbol{N}\boldsymbol{X} = \boldsymbol{H} + \widetilde{\boldsymbol{N}}$
4: Obtain $\boldsymbol{W}_k$ by solving Equ. (9) $\boldsymbol{W}_k = \frac{g_k}{g_k + \sigma^2} \boldsymbol{I}$
5: Estimate $\widehat{\boldsymbol{h}}_k$ using $\widehat{\boldsymbol{h}}_k = \boldsymbol{W}_k^H \boldsymbol{z}_k$
6: Increment in time index *i=i+1*
7: **Until** $i = N$

B. Algorithm 2 [9]

Second algorithm mentioned in [9] is based on joint pilot optimization and channel estimation whose channel estimate is obtained using $h_k = W_k^H Y u_k$, where $W_k \epsilon C^{MxM}$ and $u_k \epsilon C^{Nx1}$ are the optimization variables. Now, using the above formulation, the MMSE objective function defined in (6) can be shown to

$$\xi_k = \boldsymbol{u}_k^H \left(\textstyle\sum_{i=1}^K g_i \boldsymbol{x}_i \boldsymbol{x}_i^H + \sigma^2 \boldsymbol{I}_N\right) \boldsymbol{u}_k tr\{(\boldsymbol{W}_k^H \boldsymbol{W}_k)\} + g_k \boldsymbol{I}_M - (g_k \boldsymbol{x}_k^H \boldsymbol{u}_k) \mathrm{tr}\{\boldsymbol{W}_k^H\} - (g_k \boldsymbol{x}_k^H \boldsymbol{u}_k) tr\{\boldsymbol{W}_k\} \tag{10}$$

By differentiating the above w.r.t. W_k and setting the equation to zero, we get solution of W_k for fixed u_k and x_k.

$$\boldsymbol{W}_k = \frac{g_k \boldsymbol{x}_k^H \boldsymbol{u}_k}{\sum_{i=1}^K g_i \boldsymbol{x}_k^H \boldsymbol{u}_k \boldsymbol{u}_k^H \boldsymbol{x}_i + \sigma^2 \boldsymbol{u}_k^H \boldsymbol{u}_k} \boldsymbol{I}_M \tag{11}$$

Next, in order to optimize u_k, the following objective function is formulated

$$\min_{u_k} \widetilde{\xi}_k = \max_{u_k} \frac{\boldsymbol{u}_k^H (g_k^2 \boldsymbol{x}_k \boldsymbol{x}_k^H) \boldsymbol{u}_k}{\boldsymbol{u}_k^H (\sum_{i=1}^K g_i \boldsymbol{x}_i \boldsymbol{x}_i^H + \sigma^2 \boldsymbol{I}_N) \boldsymbol{u}_k}, \forall k \tag{12}$$

which results in

$$\widetilde{\boldsymbol{u}_k^*} = \boldsymbol{A}^{-\frac{1}{2}} g_k \boldsymbol{x}_k => \boldsymbol{u}_k^* = \boldsymbol{A}^{-\frac{1}{2}} \widetilde{\boldsymbol{u}_k^*} = \boldsymbol{A}^{-1} g_k \boldsymbol{x}_k \tag{13}$$

Where $\boldsymbol{A} = \sum_{i=1}^K g_i \boldsymbol{x}_i \boldsymbol{x}_i^H + \sigma^2 \boldsymbol{I}_N$

Finally, the optimization problem to solve becomes

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$$\min_{x_k} \frac{g_k x_k^H Q_k^{-2} x_k}{x_k^H \left(\frac{1}{P_k} I_N + g_k Q_k^{-1}\right) x_k} \tag{14}$$

The best solution of this problem is given by $\boldsymbol{x_k} = \gamma_k \boldsymbol{F}_k^{-\frac{1}{2}} \tilde{\boldsymbol{x}}$, where eigenvector $\tilde{x}$ is analogous to the largest value of the matrix $g_k \boldsymbol{F}_k^{-\frac{1}{2}} \boldsymbol{Q}_k^{-2} \boldsymbol{F}_k^{-\frac{1}{2}}$ with $\boldsymbol{F_k} = g_k \boldsymbol{Q}_k^{-1} + \frac{1}{P_k} \boldsymbol{I}_N$ and γ_k is carefully chosen so that $x_k^H x_k = P_k$ is confirmed, maximum transmission power is P_k is the at each UE .

Pseudo-code of Algorithm 2

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    Initialize weights W_k and vector u_k
Repeat
    Solve Equ. (14) and evaluate using x_k = γ_k F_k^(-1/2) x̃
    Increment in time index i=i+1
Until Convergence
Repeat
    Obtain W_k by solving Equ. (11)
    W_k = g_k x_k^H u_k / (Σ_{i=1}^{K} g_i x_k^H u_k u_k^H x_i + σ^2 u_k^H u_k) I_M
    Compute u_k by u_k^* = A^(-1) g_k x_k
    Estimate ĥ_k using ĥ_k = W_k^H Y u_k
    Increment in time index i=i+1
Until i = N
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IV. Proposed Model for the Performance Analysis

The aim of this work is to examine the influence of Log-Normal shadowing on the performance of the channel estimators described in previous sections. To do so, we incorporate the Log-Normal shadowing from [7] into the variable g_k as follows

$$g_k = (\alpha_k \beta_k)^2 \tag{15}$$

where α_k is path-loss for the kth user which can be calculated as

$$\alpha_k = \sqrt{\frac{L_k}{d_k^{\tau}}} \tag{16}$$

The power path-loss which is L_k of the link connected with kth user, the length between BS and user is repressed as d_k and the path-loss exponent τ can be chosen between 2 and 4 according to the environment. Now, the second term in Equation (15) is β_k, which is the Log-Normal shadowing parameter and it is defined as

$$\beta_k = 10^{\frac{\sigma_{\beta k} v_k}{10}} \tag{17}$$

Here, v_k is the Gaussian random variable with zero mean and unit variance and $\sigma_k$represents shadowing spread in dB. In this investigation, we will use the model defined by Equations (15)-(17) to study the following factors:

- The impact of power path loss, L_k
- The effect due to varying number of antennas M at the base station
- The effect due to varying number of users K in the system
- The performance comparison of the two channel-estimation algorithms given in Equations (9) and (11) for different values of power path loss L_k.

V. Advantages of the Proposed Model

The proposed model defined by Equations (15)-(17) can provide us several advantages over existing model which can be outlined as follows:

- It can quantify the impact of path loss via the parameter L_k
- It can provide us the effect of log-normal shadowing via the parameter β_k
- It can model the variability in the Gaussian random variable that causes the log-normal shadowing via the variance σ_k
- It can combine both the effect of large-scale fading and small-scale fading using the relation given in (16).

VI. Simulation Results

In this section, performance analyses of the two channel-estimation algorithms are deliberated with the parameters mentioned in section IV. Normalized weighted MSE (WMSE) is considered as the performance metric. In the first example, path loss L_k is selected among 0.1, 0.5 and 0.9 and results are plotted accordingly. It is clear from the result, shown in Fig. 1, that at low SNR, performance of the channel estimation is poor when path loss L_k is small. For example, there is a decrease in WMSE of 0.15 at 0 dB SNR while 0.02 at 15 dB SNR.

In the second example, we compare channel estimation performances with number of antennas at BS selected from 8 and 128. From the results, as presented in Fig. 2, it can be observed that the WSMSE of channel estimation degrades as the number of antennas at BS increases.

In the third case, performance is analyzed based on different number of users, K, in the system. The result in Fig. 3 clearly

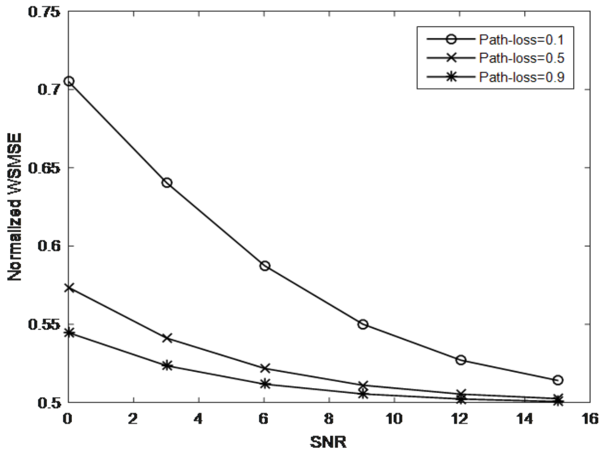

Fig. 1: Effect Path Loss (L_k)

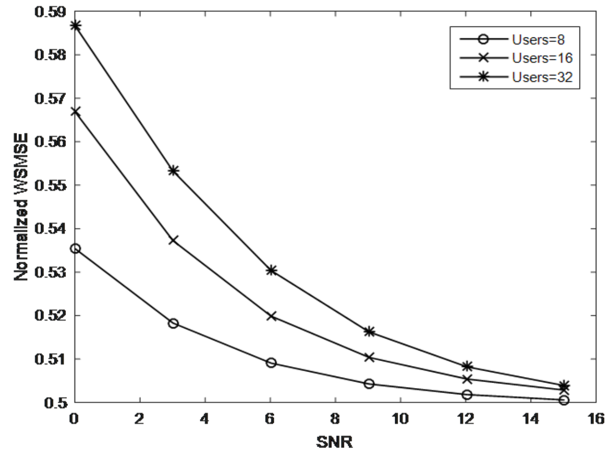

Fig. 3: Effect of Number of users

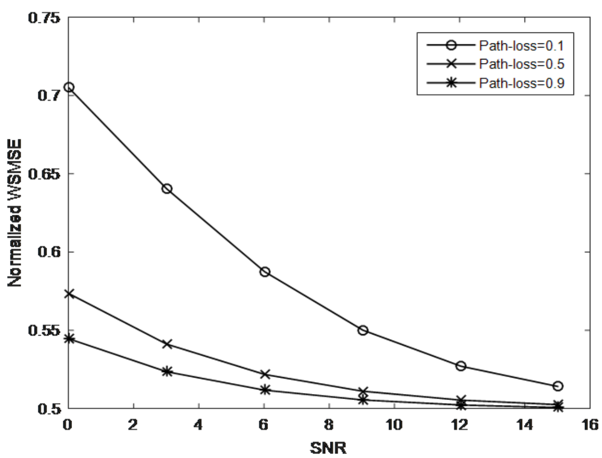

Fig. 2: Effect of Antenna Size

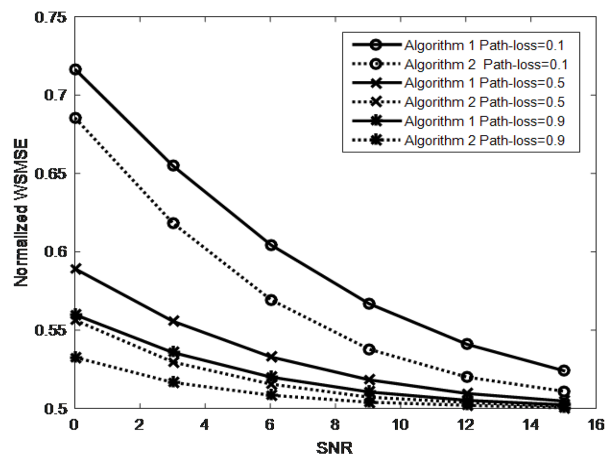

Fig. 4: Effect of power path-loss on Two Algorithms

shows that the presence of more users in the system degrades the performance of channel estimation. This is due to the fact that the increase in users enhances the impact of interference in the system. Moreover, it can be observed in the Fig. 3 that the impact of user interference is more at smaller SNR as compared to the larger SNR. For example, in going from user 8 to 32, there is a decrease of 0.05 in the WMSE at SNR of 0 dB while this decrease in WMSE is 0.01 at SNR of 12 dB.

Finally, the performance of the two estimation algorithms is analyzed for various values of the path-loss as shown in Fig. 4. The path loss L_k is selected from 0.1, 0.5, and 0.9. It is evident from the results shown in the Fig. 4 that performance of both algorithms is affected by the increase in path-loss. However, this effect is more dominant at low SNR values. Moreover, it can be seen that the decrease in the WMSE is more prominent in Algorithm 1 in contrast to Algorithm 2.

VII. CONCLUSION

An improved system model for the massive MIMO system is introduced which can incorporate the impact of both large and small scale fading. More specifically, the modified system model includes the impact of path-loss and log-normal shadowing. Performance of MMSE based two channel estimation algorithms is analyzed with the modified system model for various values of path loss, number of antennas at the base station, and number of users. The results show that the performance of channel estimation algorithms is more affected by the path loss at lower SNR values (e.g., decrease in WMSE of 0.15 and 0.02 at 0 dB and 15 dB SNRs, respectively). The results also indicate that the performance of channel estimation became poor as the number of antennas at BS increases. The effect of number of users is found to be dominant at lower SNR (e.g. decrease in WMSE is 0.05 and 0.01 at 0 dB and 12 dB SNRs, respectively). Finally, same behavior is observed for the path-loss. However, the decrease in the WMSE is more significant in Algorithm 1 as compared to Algorithm 2.

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