Analysis of Multiple Access Interference in Synchronous Downlink CDMA Systems with Random Signature Sequences in AWGN Environment

M. Moinuddin, A. U. H. Sheikh, and A. Zerguine

Abstract— In this work, analysis of the multiple access interference (MAI) for synchronous CDMA systems is carried out for the BPSK modulation with random signature sequences in additive white Gaussian noise (AWGN) environment. In this analysis, downlink scenario is considered for two different cases: (1) with perfect power control, and (2) with imperfect power control. Expressions for the probability density function (pdf) of MAI and MAI plus noise are derived for the two cases as a function of number of users and spreading factor. Gaussian approximation is also developed for these scenarios. Close agreement between analytical analysis and simulation results is obtained for different scenarios of number of users and spreading lengths.

I. Introduction

It is well known that MAI is a limiting factor in the performance of multiuser systems. Therefore, the characterization of MAI is important in the performance analysis of multiuser systems.

Two approaches for DS-CDMA, operating on AWGN channels, have been widely reported in the literature. The first approach presumes that exact BER evaluation is intractable or numerically cumbersome, so accurate bit-error rate (BER) approximations are sought [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. Perhaps the most widely cited and most widely used approximation is the so-called standard Gaussian approximation (SGA) [1]-[3], [6]-[11]. In the SGA, a central limit theorem (CLT) is employed to approximate the sum of the multiple-access interference (MAI) signals as an additive white-Gaussian process additional to the background Gaussian noise process. The receiver design, thus, consists of a conventional single-user matched filter (correlation receiver) to detect the desired user signal. The average variance of the MAI over all possible operating conditions is used to compute the signal-to-noise ratio (SNR) at the filter (correlator) output. The SGA is widely used because it is easy to apply; however, it is observed that performance analysis based on using the SGA often overestimate the system performance, especially when the number of users in the system is small [4]. These limitations have motivated research to improve the accuracy of the SGA. In [9], the accuracy of the SGA was improved by using the standard Hermite polynomial error correction method. In [12], the statistics of the MAI signals with random signature sequences were extensively studied. Based on the work of [12], [4] later introduced a method termed improved Gaussian approximation (IGA). The IGA is more accurate than the SGA, especially for a small number of active users [4]. However, the IGA computation requires numerical integration and multiple numerical convolutions. This method was simplified in [5] such that neither knowledge of the conditional variance distribution, nor numerical integration, nor convolution is necessary to achieve acceptable BER estimation. Thus, it is termed simplified IGA (SIGA) [5]. Later, Morrow [11] further simplified the expression attained in [5] without significant penalty in the BER accuracy. More recently, based on the work of [5], Young C. Yoon gave a generalized simplified improved gaussian approximation that can be applied to band-limited pulse shapes as well as general pulse shapes [14].

The second approach is to perform the evaluation of the CDMA system BER without any knowledge of or assumption about the MAI distribution. Many of these techniques are based on extensions of previous studies of inter-symbol interference (ISI) systems. These methods include the moment space technique [13], characteristic function (CF) method [15], method of moments [16], and an approximate Fourier series method [17]. In [15], Geraniotis and Pursley used the CF method to evaluate CDMA system performance in an AWGN channel. Generally, these techniques can achieve more accurate BER estimate than CLT-based approximations at the expense of much higher computational complexity.

In this work, analysis of MAI for synchronous CDMA systems is carried out for BPSK with random signature sequences in AWGN channel using a novel approach. The main contribution of the work presented is that analysis is carried out without using

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the Gaussian approximation for MAI. Consequently, new closed form expressions for the MAI are derived for both perfect power control and imperfect power control scenarios.

The paper is organized as follows. Following this introduction, the system model considered for the analysis is presented in Section II. The receiver decision statistic is obtained in Section III and the pdf of MAI is obtained in Section IV. Simulation results are presented in Section VI. Finally, the paper is concluded in Section VII.

II. System Model

A synchronous DS-CDMA transmitter model for the downlink of a mobile radio network is considered as shown in Figure (1). If there are K users in the

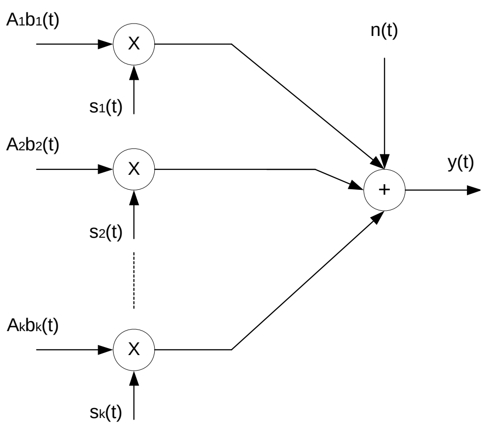

Fig. 1. System Model in AWGN

system, then the signal at the receiver input is given by

$$y(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} A_k b_{i,k} s_{i,k}(t) + n(t) \qquad (1)$$

where

- $s_{i,k}(t)$ is the random signature waveform of the kth user defined in $iT_b \leq t \leq (i+1)T_b$. $s_{i,k}(t)$ is normalized to have unit energy i.e.,

$$\int_{iT_b}^{(i+1)T_b} |s_{i,k}(t)|^2 dt = 1 \qquad (2)$$

The signature waveform of length $N_c = T_b/T_c$ is defined as

$$s_{i,k}(t) = \sum_{j=1}^{N_c} c_{i,k,j} rect\Big(\frac{t + .5T_c - jT_c}{T_c}\Big) \qquad (3)$$

for $(j-1)T_c \leq t \leq jT_c$ where T_b and T_c are bit period and chip interval, respectively. $\{c_{i,k,j}\}$ is the normalized spreading sequence of user k for the i^{th} symbol which takes the values $\{-1,+1\}$ with equal probability. $rect(t)$ is the rectangular chip waveform defined as:

$$rect\Big(\frac{t - .5T_c}{T_c}\Big) = \begin{cases} 1, & 0 \leq t \leq T_c \\ 0, & \text{elsewhere} \end{cases}$$

- $\{b_{i,k}\}$ is the input bit stream of the k^{th} user having support $\{b_{i,k}\} \in \{-1,+1\}$.
- A_k is the transmitted amplitude of the k^{th} user.
- $n(t)$ is the additive white gaussian noise with variance σ_n^2.

Since synchronous CDMA is considered, it is assumed that the receiver has some means of achieving perfect chip synchronization. The cross-correlation of the signature sequences are defined as

$$\rho_{i,kj} = \int_{(i-1)T_b}^{iT_b} s_{i,k}(t) s_{i,j}(t) dt = \sum_{m=1}^{N_c} c_{i,k,m} c_{i,j,m} \qquad (4)$$

III. Receiver Decision Statistic

In conventional single-user digital communication systems, the matched filter is used to generate sufficient statistics for signal detection. This matched filter is matched to the signature waveform of the desired user (say user 1). It is worth mentioning that we need exact knowledge of the desired user's signature sequence and the signal timing in order to implement this detector. Without loss of generality, we can assume the desired user to be user 1. Thus, the output of the matched filter for the i^{th} symbol is equivalent to the correlator's output sampled at symbol intervals, that is,

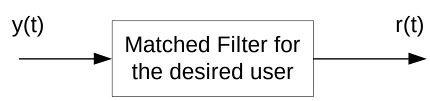

Fig. 2. Matched Filter matched to the signature waveform of the desired user

$$r_i = \int_0^{T_b} y_i(t) s_{i,1}(t) dt \qquad (5)$$

Upon substituting the value of $y_i(t)$ from equ. (1) and knowing that $\rho_{i,11} = 1$, the output of the matched filter for the i^{th} symbol is found to be

$$r_i = A_1 b_{i,1} + \sum_{k=2}^{K} A_k b_{i,k} \rho_{i,k1} + n_i \qquad (6)$$

The above equation will serve as a basis for our analysis, especially the second term which corresponds to the MAI. In the above, the first term $(A_1 b_{i,1})$ is the desired user's data while the last term is due to the additive noise.

IV. The Pdf of MAI and MAI plus Noise

Denoting the MAI term by M, the MAI for the i^{th} symbol can be represented as

$$M_i = \sum_{k=2}^{K} A_k b_{i,k} \rho_{i,k1} \tag{7}$$

Let us define a new random variable X_k such that

$$X_k \stackrel{\triangle}{=} b_{i,k} \rho_{i,k1} \tag{8}$$

Thus, it can be observed that the value of the random variable X_k ranges between 1 and -1. Consequently, we can set up X_k as follows:

$$X_k = (N_c - 2d)/N_c, \quad d = 0, 1, \cdots, N_c \tag{9}$$

where that d is a binomial random variable with equal probability of success and failure, i.e.,

$$P_D(d = r) = \binom{N_c}{r} \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{N_c - r}$$

If the interference from k^{th} interfere is denoted by I_k, then we can set up the MAI as

$$M_i = \sum_{k=2}^{K} I_k \tag{10}$$

where

$$I_k = A_k X_k \tag{11}$$

The characteristic function of a discrete random variable with point mass function (PMF) $P_X(x_i)$ is given by:

$$\Phi_X(\omega) = \sum_i e^{j\omega x_i} P_X(x_i) \tag{12}$$

Thus, the characteristic function of the discrete random variable I_k can be obtained by substituting $x_i = A_k(N_c - 2i)/N_c$ and found to be

$$\begin{aligned}\Phi_{I_k}(\omega) &= \sum_{i=0}^{N_c} e^{j\omega A_k (N_c - 2i)/N_c} \binom{N_c}{i} \left(\frac{1}{2}\right)^{N_c} \\ &= e^{j\omega A_k} \left(\frac{e^{-2j\omega A_k/N_c} + 1}{2}\right)^{N_c} \end{aligned} \tag{13}$$

We know that that the characteristic function of a sum of random variables is the product of their individual characteristic functions. Thus, the characteristic function of MAI is given by:

$$\begin{aligned}\Phi_M(\omega) &= \prod_{k=2}^{K} \Phi_{I_k}(\omega) \\ &= e^{j\omega \sum_{k=2}^{K} A_k} \prod_{k=2}^{K} \frac{e^{-2j\omega A_k/N_c} + 1}{2}^{N_c} \end{aligned} \tag{14}$$

Next we discuss the two cases depending on the scenarios of power control.

A. Case 1: Scenario of Perfect Power Control

When there is a perfect power control, i.e., all the users have equal power, that is,

$$A_k = 1, \quad \forall\, k \tag{15}$$

As a result, the characteristic function of MAI will become

$$\Phi_M(\omega) = e^{j\omega(K-1)} \frac{e^{-2j\omega/N_c} + 1}{2}^{(K-1)N_c} \tag{16}$$

If the additive noise term n_i, in equation (6), is $N(0, \sigma_n^2)$, then its characteristic function is given by:

$$\Phi_N(\omega) = e^{-\sigma_n^2 \omega^2/2} \tag{17}$$

If Z_i denotes the MAI plus noise term, its characteristic function, $\phi_{Z_i}(\omega)$, is given by:

$$\begin{aligned}\phi_{Z_i}(\omega) &= \phi_{MAI_i}(\omega)\phi_N(\omega) \\ &= e^{j\omega(K-1)} \frac{e^{-2j\omega/N_c} + 1}{2}^{(K-1)N_c} e^{-\sigma_n^2 \omega^2/2} \end{aligned} \tag{18}$$

Eventually, the pdf of MAI plus noise is given by:

$$\begin{aligned}f_{Z_i}(z_i) &= \frac{1}{\sqrt{2\pi\sigma_n^2}} \sum_{l=0}^{(K-1)N_c} \binom{(K-1)N_c}{l} \\ &\quad \times \left(\frac{1}{2}\right)^{(K-1)N_c} e^{-\frac{(z_i - (K-1) + 2l/N_c)^2}{2\sigma_n^2}} \end{aligned} \tag{19}$$

B. Case 2: Scenario of Imperfect Power Control

When there is imperfect power control, the users have different powers, that is,

$$A_j \neq A_k, \quad \forall\, j, k \tag{20}$$

Consequently, the characteristic function of MAI is given by the equation (14). Now, we will investigate the behavior of a single interferer.

We can set up the noise term n_i in equation (6) as the sum of $(K-1)$ terms each having zero mean and equal variance $\sigma_{n,k}^2 = \sigma_n^2/(K-1)$. Hence, we can rewrite the sum of MAI and noise as follows:

$$\begin{aligned} Z_i &= MAI_i + n_i \\ &= \sum_{k=2}^{K} I_k + \sum_{k=2}^{K} n_k \\ &= \sum_{k=2}^{K} I_k^{'} \end{aligned} \tag{21}$$

where $I_k^{'} = I_k + n_k$. Thus the characteristic function of $I_k^{'}$, i.e., $\phi_{I_k^{'}}(\omega)$ is given by:

$$\begin{aligned}\phi_{I_k^{'}}(\omega) &= \phi_{I_k}(\omega)\phi_{n_k}(\omega) \\ &= e^{j\omega A_k} \sum_{l=0}^{N_c} \binom{N_c}{l} \left(\frac{e^{-2j\omega A_k/N_c}}{2}\right)^{l} \\ &\quad \times \left(\frac{1}{2}\right)^{N_c - l} e^{-\sigma_{n,k}^2 \omega^2/2} \end{aligned} \tag{22}$$

Finally, the pdf of the effective single interferer ($I_k^{'}$) is obtained by inverse transform of characteristic function as follows:

$$\begin{aligned} f_{I_k^{'}}(z_i) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{I_k^{'}}(\omega)e^{-j\omega z_i}d\omega \\ &= \frac{1}{\sqrt{2\pi\sigma_{n,k}^2}}\sum_{l=0}^{N_c}\binom{N_c}{l}\left(\frac{1}{2}\right)^{N_c} \\ &\quad \times e^{-\frac{(z_i-A_k+2A_k l/N_c)^2}{2\sigma_{n,k}^2}} \end{aligned} \quad (23)$$

V. Gaussian Approximations for MAI

In this section, we have developed Gaussian approximations for the pdf of MAI plus noise. It is a known fact that pdf Gaussian distributed random variable is completely characterized via two parameters, i.e., mean and variance of the random variable. Thus, in order to derive the Gaussian approximation, we will first evaluate the mean and the variance of the desired random variable for both the cases of perfect and imperfect power control.

The MAI at the output of the matched filter is given by equation (7). Using equation (8), we can rewrite the equation (7) as follows:

$$MAI_i = \sum_{k=2}^{K} A_k X_k \quad (24)$$

where X_k is defined by the equation (9) such that d is a binomial random variable with equal probability of success and failure. Thus, we have

$$E[d] = \frac{1}{2}N_c \quad \text{and} \quad \sigma_d^2 = \frac{1}{4}N_c \quad (25)$$

As a result, mean of MAI (μ_m) is given by:

$$\mu_m = \sum_{k=2}^{K} A_k(1 - \frac{2}{Nc}E[d]) = 0 \quad (26)$$

Assuming each interferer to be independent with zero mean, the variance of MAI (σ_m^2) is given by:

$$\begin{aligned} \sigma_m^2 &= \sum_{k=2}^{K} E[A_k^2 X_k^2] \\ &= \sum_{k=2}^{K} A_k^2 E[(1-\frac{2}{Nc}d)^2] \\ &= \sum_{k=2}^{K}\frac{A_k^2}{N_c} \end{aligned} \quad (27)$$

A. Case 1: Scenario of Perfect Power Control

In case of perfect power control, the transmitted signal power of all the users are equal, so without loss of generality we can assume $A_k = 1 \;\forall\; k$. Thus, using equation (27), the variance of MAI with perfect power control is given by:

$$\sigma_m^2 = \frac{(K-1)}{N_c} \quad (28)$$

Hence, Gaussian approximation for the pdf of MAI is given by:

$$f_{M_i}(m_i) = \frac{1}{\sqrt{2\pi\left(\frac{(K-1)}{N_c}\right)}}\, exp\left[\frac{-m_i^2}{2\left(\frac{(K-1)}{N_c}\right)}\right] \quad (29)$$

Since, the additive noise is independent with MAI and has zero mean and variance σ_n^2, thus, the mean and the variance of overall MAI plus noise term (Z_i) are given by

$$\begin{aligned} \mu_z &= 0 \\ \text{and} \quad \sigma_z^2 &= \sigma_m^2 + \sigma_n^2 \\ &= \frac{(K-1)}{N_c} + \sigma_n^2 \end{aligned} \quad (30)$$

Hence, Gaussian approximation for the pdf of Z_i is given by:

$$f_{Z_i}(z_i) = \frac{1}{\sqrt{2\pi\left(\frac{(K-1)}{N_c} + \sigma_n^2\right)}}\, exp\left[\frac{-z_i^2}{2\left(\frac{(K-1)}{N_c} + \sigma_n^2\right)}\right] \quad (31)$$

B. Case 2: Scenario of Imperfect Power Control

In case of imperfect power control, the transmitted signal power of users are unequal, i.e. $A_j \neq A_k, \;\; \forall\; j,k$. Thus, the variance of MAI with perfect power control is given by equation (27). Thus, the Gaussian approximation for the pdf of MAI is given by:

$$f_{M_i}(m_i) = \frac{1}{\sqrt{2\pi\left(\sum_{k=2}^{K}\frac{A_k^2}{N_c}\right)}}\, exp\left[\frac{-m_i^2}{2\left(\sum_{k=2}^{K}\frac{A_k^2}{N_c}\right)}\right] \quad (32)$$

the mean and the variance of overall MAI plus noise term (Z_i) are given by

$$\begin{aligned} \mu_z &= 0 \\ \text{and} \quad \sigma_z^2 &= \sigma_m^2 + \sigma_n^2 \\ &= \sigma_n^2 + \sum_{k=2}^{K}\frac{A_k^2}{N_c} \end{aligned} \quad (33)$$

Hence, the Gaussian approximation for the pdf of Z_i is given by:

$$f_{Z_i}(z_i) = \frac{1}{\sqrt{2\pi\left(\sigma_n^2 + \sum_{k=2}^{K}\frac{A_k^2}{N_c}\right)}} exp\left[\frac{-z_i^2}{2\left(\sigma_n^2 + \sum_{k=2}^{K}\frac{A_k^2}{N_c}\right)}\right] \quad (34)$$

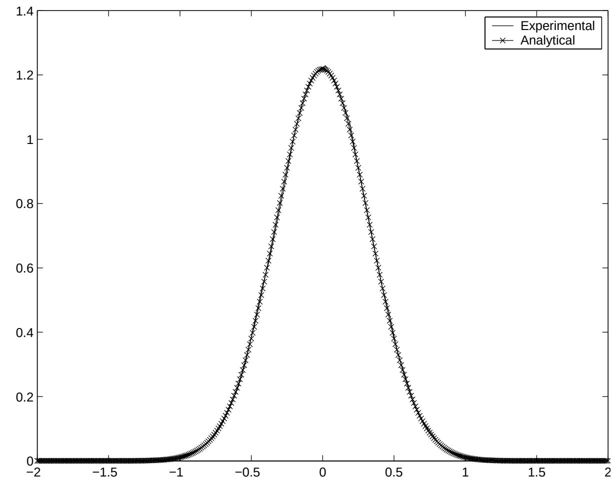

Fig. 3. Comparison between Analytical and Experimental results for the pdf of MAI plus noise with Perfect Power Control for 4 users

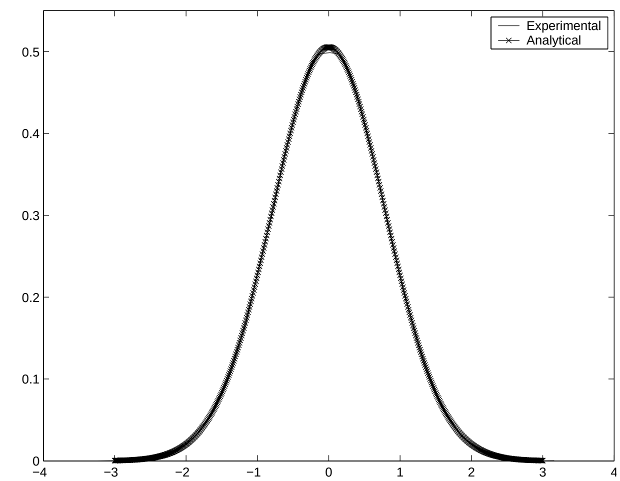

Fig. 4. Comparison between Analytical and Experimental results for the pdf of MAI plus noise with Perfect Power Control for 20 users

VI. Simulation Results

Simulation results are presented in this section to validate the theoretical findings. The pdf for MAI plus noise derived for the two different cases, i.e., with perfect and imperfect power control, given by equations (19) and (23), respectively, are investigated via simulations. These analytical results are compared with the simulation one.

In simulation, we have used random signature sequences of length 31 and chip waveforms are rectangular. Signal to noise ratio is kept 20 dB for all the cases. Figures 3, and 4 show the comparison of experimental and analytical results for the pdf of MAI plus noise with perfect power control for 4 and 20 users, respectively. The results show that the overall behavior of MAI plus noise in AWGN with perfect power control is normal distributed. As can be seen from these figures, close agreement between analytical result and simulation is obtained. Also, it is obvious from these figures that the variance of MAI increases with the increase in number of users which is verifying the theoretical result.

Next, simulation is carried out to investigate the accuracy of the Gaussian approximations derived for MAI which are given by equations (29) and (32) with perfect and imperfect control, respectively. Figures 5, and 6 show the Gaussian approximated pdf of MAI plus noise with perfect power control for 4 and 20 users, respectively. The results show the validation of Gaussian approximation. Similarly, Figure 7 shows the pdf of single interferer with perfect power control for 4 users verifying the use of Gaussian approximation. It can be inferred from the result of single interferer that overall MAI will also be Gaussian.

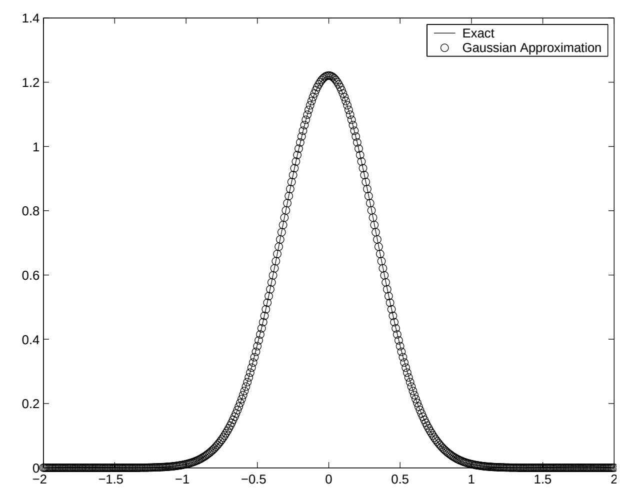

Fig. 5. Comparison between Exact and Gaussian approximation for the pdf of MAI plus noise with Perfect Power Control for 4 users

VII. Conclusion

This work has presented the investigation of MAI in synchronous CDMA systems is for the BPSK modulation with random signature sequences in additive white Gaussian noise channel. As a result, closed form expressions for the pdf of MAI and MAI plus noise are derived for both perfect and imperfect power control scenarios. It is shown that the pdf of MAI and MAI plus noise is a function of the number of transmitted users and the spreading factor. Gaussian approximation is also developed for these pdfs which are then compared with the exact pdfs. Simulation results presented support our theoretical analysis.

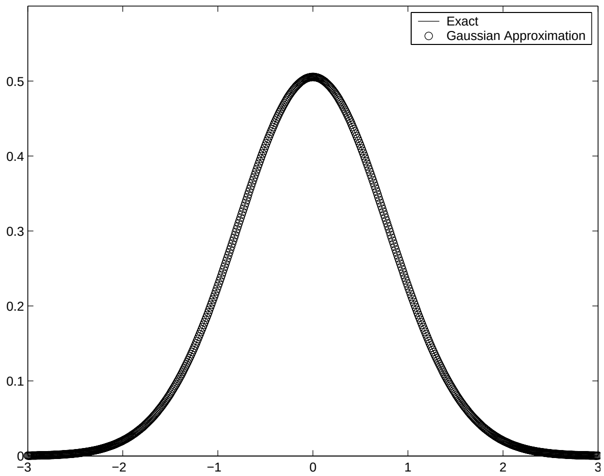

Fig. 6. Comparison between Exact and Gaussian approximation for the pdf of MAI plus noise with Perfect Power Control for 20 users

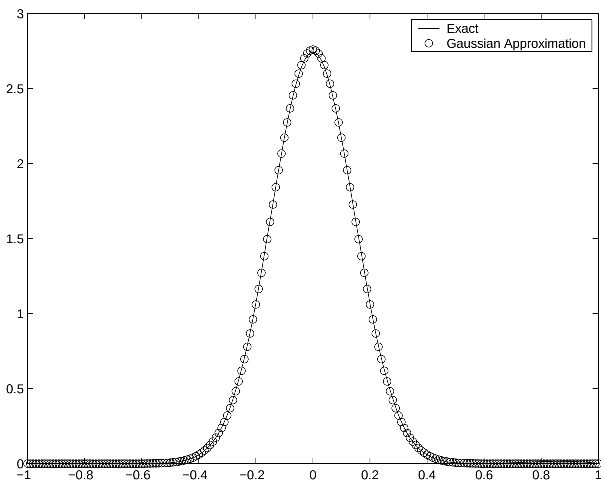

Fig. 7. Comparison between Exact and Gaussian approximation for the pdf of MAI plus noise with Imperfect Power Control for 4 users

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