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A Comparative Study of MIMO-DFE Receivers

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***Abstract*—A decision feedback equalizer (DFE) is a nonlinear equalizer which utilizes the previous detector assessment to mitigate the intersymbol interference (ISI) on received symbols. Alternatively, we can say that the distortion on the current symbol that was caused by previous symbols is eliminated. While linear equalizer (LE) can be employed on the channels where the ISI is not significantly present but in case when there is a severe ISI channel then DFE's have been effectively used to mitigate ISI. Different techniques have been proposed to improve the decision feedback equalization performance. In this paper we have reviewed some of the existing equalization techniques and have highlighted their performance gains.**

***Index Terms*—DFE, MIMO, Equalization, LMS, Constrained optimization, MMSE.**

I. Introduction

The MIMO communication schemes offer the potential for significant increases in spectral efficiency over their single-input single output counterparts by enabling simultaneous transmission of independent data streams. MIMO schemes also offer the potential for significant performance gains in a variety of other metrics. However, equalization of wireless MIMO frequency-selective channels is a challenging task mainly due to the fact that the respective MIMO equalizers should cope with ISI as well as inter-user interference (IUI).

The development of the DFE was initiated by the idea of using previous detected symbols to compensate for the ISI in a dispersive channel. In recent years, there has not been much development on the structure of the linear and decision feedback equalizers. However considerable effort has been given to the investigation of adaptive algorithms that are used to adapt the equalizers according to the prevalent CIR. In the same context, one constrained in DFE is that the decision errors fed back to the FBF may effect the performance of the DFE. Therefore, it is required to mitigate decision errors to improve its performance.

As implied by the terminology, the decision feedback equalizer (DFE) employs a feedfordward filter and feedback filter in order to combat the ISI inflicted by the dispersive channels. The non-linear function manifested by the decision device is introduced at the input of the feedback filter. The general block diagram of the DFE is shown in Figure (1). In general, as in the linear MSE equalizer, the forward filter partially eliminates the ISI introduced by the dispersive channel. The feedback filter, in the absence of decision errors, is fed with the error-free signal in order to further eliminate the ISI.

In MIMO systems with relatively long bursts and under time varying conditions, the involved channel impulse responses

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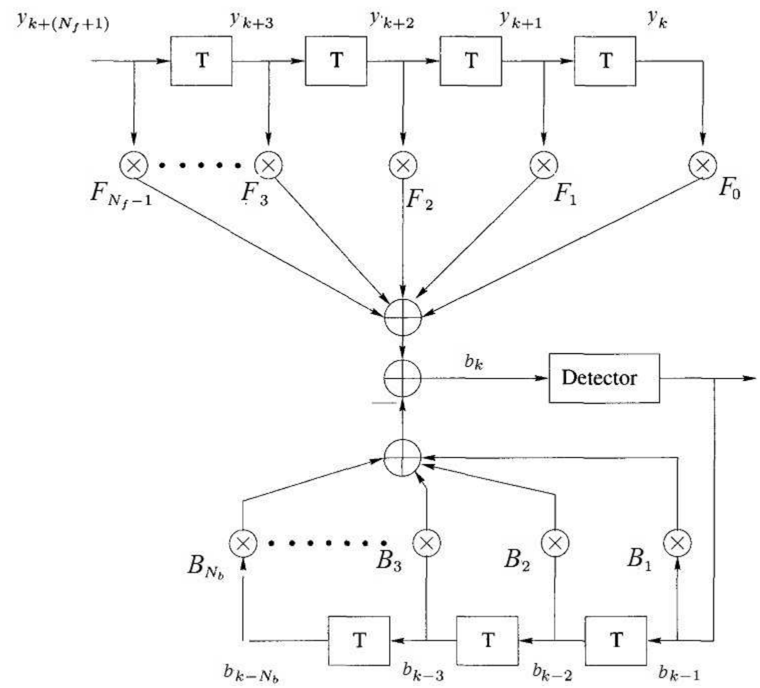

Fig. 1. Decision feedback equalizer structure

change within a burst and, as expected, batch MIMO DFEs fail to equalize the channel. On the other hand, if a MIMO OFDM is adopted, the frame size should be made short and thus cyclic prefix overhead becomes overwhelming. Therefore, to achieve effective channel equalization in such cases, adaptive methods are required. Both a minimum bit error rate (MBER) design [1], [2] and the standard minimum mean-square error (MMSE) design [3] have been invoked for implementing adaptive MIMO DFEs. The respective equalizers are updated either by using gradient Newton methods (i.e., RLS, LMS-Newton) or by employing stochastic gradient techniques. The main problems appearing in adaptive MIMO equalization, i.e., the increased filter size and the colored noise caused by inter-stream interference, slow down significantly the performance of stochastic gradient algorithms. On the other hand, the computational requirements of MIMO RLS algorithms increase significantly. In [4], some adaptive schemes with convergence properties close to RLS but of lower computational cost are proposed. But still there computational complexity is very high compared to the LMS type algorithm. For scenarios in which accurate channel state information (CSI) is available at both the transmitter and the receiver, there is a well established framework that unifies the design of linear transceivers under many design criteria [5]. A counterpart for the design of systems with DFE has recently emerged [6], [7]. This framework was also extended to MIMO systems with pre-interference subtraction at the transmitter in [6].

In this paper, we have reviewed some of the MIMO-DFE receiver techniques that have been employed to improve its performance. This paper is organized as follows: Following the introduction, the second section describes the system model. The third section outlines the different methodologies proposed in the respective papers. Performance enhancements achieved with the techniques are discussed in the fourth section. Some conclusive remarks are presented in the fifth and finally some future research prospects are highlighted in the last section.

II. System Model

The system model consists of a DFE, depicted in Figure (2), with finite number of taps in the FFF and FBF, denoted by $\mathbf{F}$ and $\mathbf{B}$, respectively. For a MIMO system, assuming there are M transmit and N receive antennas, respectively, we define

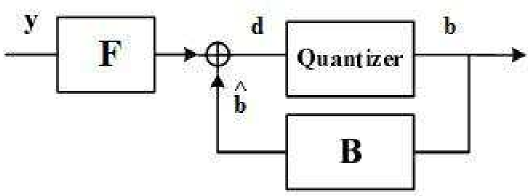

Fig. 2. Decision feedback equalizer

$$\mathbf{b}(k) = \left[b_1(k)\ b_2(k) \ldots b_M(k)\right]^T, \quad (1)$$
$$\mathbf{y}(k) = \left[y_1(k)\ y_2(k) \ldots y_N(k)\right]^T. \quad (2)$$

as the transmitted and received signal vectors, respectively. We denote $h_{nm}(p)$ as the p^{th} multipath coefficient of the MIMO channel from transmit antenna m to receive antenna n. We also assume that all the channels from each transmit to receive antenna have finite impulse response of the same length P. Now let the matrix $[H(p)]_{nm} = h_{nm}(p)$. Then, we can write

$$\mathbf{y}(k) = \sum_{p=0}^{P-1} H(p)\,\mathbf{b}(k-p) + \mathbf{z}(k), \quad (3)$$

where $\mathbf{z}(k) = \left[z_1(k)\ z_2(k) \ldots z_N(k)\right]^T$ is the Gaussian noise vector with zero mean and variance σ_z^2. If we let L be the length of the FFF, we can define

$$\mathbf{b}_k = \left[\mathbf{b}^T(k)\ \mathbf{b}^T(k-1) \ldots \mathbf{b}^T(k-L-P+2)\right]^T,$$
$$\mathbf{y}_k = \left[\mathbf{y}^T(k)\ \mathbf{y}^T(k-1) \ldots \mathbf{y}^T(k-L+1)\right]^T,$$
$$\mathbf{z}_k = \left[\mathbf{z}^T(k)\ \mathbf{z}^T(k-1) \ldots \mathbf{z}^T(k-L+1)\right]^T.$$

Then we can write (3) as

$$\mathbf{y}_k = \mathbf{H}\mathbf{b}_k + \mathbf{z}_k, \quad (4)$$

where $\mathbf{H}$ represents the channel convolution matrix of size $[LN \times (L+P-1)M]$ and is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}(0) & \cdots & \mathbf{H}(P-1) & & \\ & \ddots & \cdots & \ddots & \\ & & \mathbf{H}(0) & \cdots & \mathbf{H}(P-1) \end{bmatrix}. \quad (5)$$

III. Receiver Design Techniques

In this section, we are going to review some of the techniques that have been employed in MIMO-DFE receivers.

A. Constrained MMSE-DFE

As mentioned earlier, the error propagation associated with DFE effects its performance. Several techniques have been devised to mitigate this constraint. One such technique proposed in [12] imposes an inequality norm constraint on the tap energy of the FBF. Therefore, the constraint on the FBF is given as

$$tr\left\{\mathbf{B}^H\mathbf{B}\right\} \leq \xi, \quad (6)$$

where ξ is the energy threshold. This results in the constraint MMSE solution for the FFF and FBF, respectively as

$$\mathbf{F}_{op}(\lambda) = \left(\mathbf{H}_1\mathbf{H}_1^H + \lambda\mathbf{H}_2\mathbf{H}_2^H + \sigma_z^2\mathbf{I}\right)^{-1}\mathbf{H}_1\mathbf{1}_1, \quad (7)$$
$$\mathbf{B}_{op}(\lambda) = \mathbf{H}_2^H\mathbf{F}_{op}(\lambda), \quad (8)$$

where $\mathbf{H}_1$ and $\mathbf{H}_2$ are the sub-matrices of $\mathbf{H}$ and $\mathbf{1}_1 = \left[\mathbf{0}\,\mathbf{0} \ldots \mathbf{0}\,\mathbf{I}_M\right]^T$ is a $(DM \times M)$ matrix; $\mathbf{0}$ is a $(M \times M)$ matrix of all zeros. The choice of the value of the multiplier λ, governs the behaviour of the constrained DFE. When $\lambda = 0$, it results in the conventional DFE. This assumes the ideal operation of the FBF, i.e., the post-cursor ISI is cancelled by the FBF. When $\lambda = 1$, it results in the MMSE linear equalizer. This assumes that the FFF will mitigate not only the precursor ISI but the post-cursor ISI as well. Severe error propagation can be dealt with by choosing λ very close to 1. The model of the DFE used is given in Figure (3).

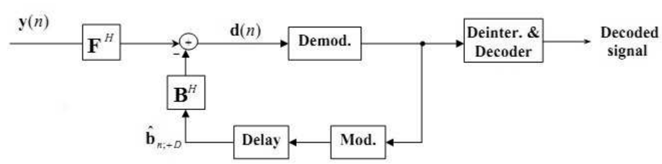

Fig. 3. Constrained MMSE-DFE

B. Cyclic MMSE

A property of modulated signals is that they are poly-cyclostationary (PCS) in nature meaning modulated signals have correlation functions which are time poly-periodic. This property of PCS together with MMSE is used to improve the equalization of modulated signals in [10]. The DFE structure, shown in Figure (4) is built from the received signal that are possibly shifted by its cyclic frequencies.

This structure is particularly beneficial for OFDM where the DFE has been shown exhibit improved BER performance in the presence of narrow band and wide band interference.

C. Limited Feedback ZF-DFE

Precoding is one technique used in digital communication to suppress the channel impairments to the transmitted symbol. A decision feedback equalizer employing this technique is shown in Figure (5).

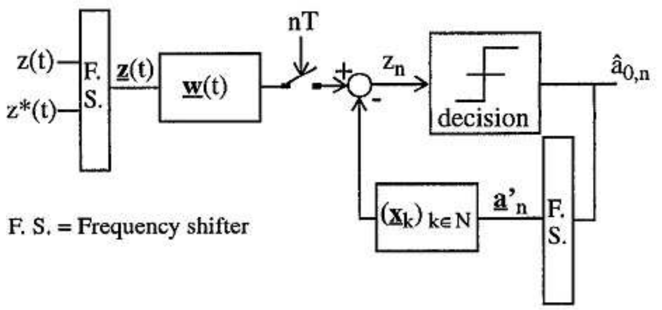

Fig. 4. Cyclic MMSE-DFE

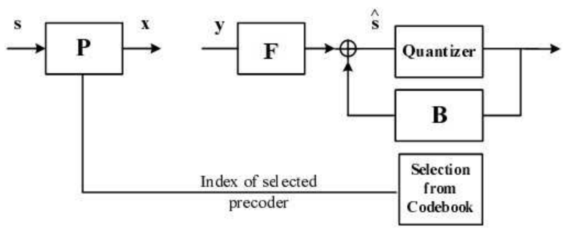

Fig. 5. Limited feedback MIMO ZF-DFE

If the channel state information (CSI) is available to the transmitter, the data can be precoded before transmission to minimize the channel effects on the symbols. This is equivalent as stating that the data is weighted at the transmitter such that the signal power is maximised at the receiver output, i.e.,

$$\mathbf{x} = \mathbf{P}\mathbf{b}, \tag{9}$$

where $\mathbf{P}$ is the precoding matrix. The approach devised in [9] presents a design framework for a limited low-rate feedback of the CSI to the transmitter with a zero-forcing DFE. This approach assumes perfect CSI at the receiver. The optimum zero-forcing DFE solution obtained can be written as

$$\mathbf{F} = \mathbf{C}\left(\mathbf{H}\mathbf{P}\right)^{\dagger}, \tag{10}$$

$$\mathbf{B} = diag\left(\mathbf{L}_{11}, \ldots \mathbf{L}_{KK}\right)\mathbf{L}^{-1} - \mathbf{I}, \tag{11}$$

where $\mathbf{C} = \mathbf{I}+\mathbf{B}$, $\mathbf{L}$ is a lower triangular matrix resulting from the Cholesky decomposition of $\sigma_z^2\left(\mathbf{P}^H\mathbf{H}^H\mathbf{H}\mathbf{P}\right)^{-1} = \mathbf{L}\mathbf{L}^H$ and $(.)^{\dagger}$ is the pseudo-inverse.

D. Adaptive Channel Aided DFE

Another approach devised in [11] uses a basic property of the DFE, i.e., the post-cursors of the channel response convolved with the FFF is canceled by the FBF. This approach results in effectively mitigating the propagation errors. The LMS algorithm is used to estimate the channel response and subsequently the optimum solution of the FBF. It incorporates the channel estimator in the DFE structure as shown in Figure (6). The update equation for the MIMO DFE can be written as

$$\mathbf{f}_{nm,k+1} = \mathbf{f}_{nm,k} + \mu_f \mathbf{y}_{nm,k} e_k^*, \tag{12}$$

where μ_f is the learning rate of the algorithm and e_k^* is the error. If the coefficients of the channel estimator is denoted by

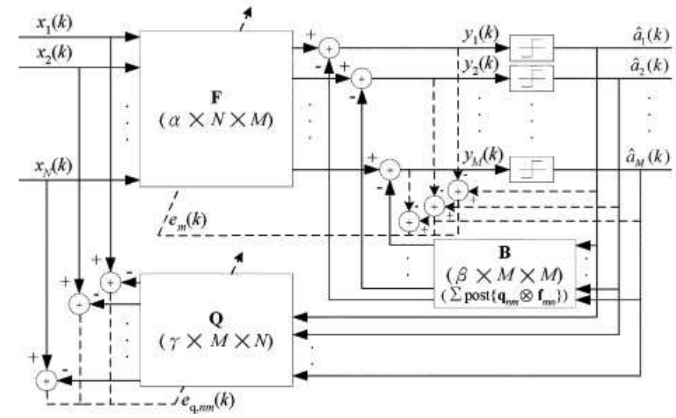

Fig. 6. Adaptive channel aided DFE

$\mathbf{q}$ then the LMS update equation for the estimator is given as

$$\mathbf{q}_{nm,k+1} = \mathbf{q}_{nm,k} + \mu_q \mathbf{b}_{nm,k} e_k^*, \tag{13}$$

where μ_q is the learning rate of the algorithm and $\mathbf{b}_k$ is the FFF decision. It is shown in [11] that this is essentially a system identification problem where $\mathbf{q}_{nm,opt} = h_{nm}^*$. With the optimum solution $\mathbf{q}_{nm,opt}$, the optimum solution for the FBF is given as

$$\mathbf{b}_{nm,opt} = \text{post}\left\{\mathbf{q}_{nm,opt} \otimes \mathbf{f}_{nm,opt}\right\}, \tag{14}$$

where $\otimes$ indicates convolution and post$\{.\}$ denotes the post-cursor-taking operation.

E. Adaptive Conjugate Gradient DFE

The complexity involved in the design of the DFE is the main issue addressed in [8]. The proposed idea applied an adaptive modified conjugate gradient algorithm to derive an equalizer with identical convergence, improved tracking capabilities but with a problem of higher computational load as compared to Recursive Least Square (RLS) algorithm. Two updating strategies of the equalizers filters based on the Galerkin projection are utilized to reduce the complexity. It has been shown that single-input single-output (SISO) adaptive algorithms based on conjugate gradient methods are numerically steady. Another advantage is that their convergence properties are comparable to the RLS and that their computational cost is in between RLS and LMS algorithms. The main motivation for this approach was that no work has been done for developing MIMO adaptive equalization algorithms based on CG method. The MIMO DFE solution using the least squares criterion can be computed as the minimum of the cost function

$$J\left(\mathbf{w}, \Phi\left(k\right), \mathbf{r}_i\left(k\right)\right) = \frac{\mathbf{w}^H\Phi\left(k\right)\mathbf{w}}{2} - Re\left\{\mathbf{w}^H\mathbf{r}_i\left(k\right)\right\} \tag{15}$$

with respect to $\mathbf{w}$. Matrix $\Phi\left(k\right)$ stands for the exponentially time-averaged input data autocorrelation and $\mathbf{r}_i\left(k\right)$ is the crosscorrelation vector. The modified CG method for the MIMO DFE minimizes the cost function in (15) by iteratively updating the vector $\mathbf{W}$ as

$$\mathbf{W}\left(k\right) = \mathbf{W}\left(k-1\right) + \mathbf{U}\left(k\right)\mathbf{A}\left(k\right), \tag{16}$$

where the columns of $\mathbf{U}(k)$ are the search direction for each M systems and $\mathbf{A}(k)$ is a $M \times M$ diagonal matrix having the ith step size, $\alpha_i(k)$ given by

$$\alpha_i(k) = \frac{\mathbf{u}_i^H(k)\,\mathbf{t}_i(k)}{\mathbf{u}_i^H(k)\,\Phi(k)\,\mathbf{u}_i(k)}, \quad i = 1, \ldots, M \tag{17}$$

The search direction is updated as

$$\mathbf{U}(k+1) = \mathbf{G}(k) + \mathbf{U}(k)\,\beta(k), \tag{18}$$

where $\mathbf{G}(k)$ is the gradient of the system given by

$$\mathbf{G}(k) = \mathbf{T}(k) - \Phi(k)\,\mathbf{U}(k)\,\mathbf{A}(k), \tag{19}$$

where $\mathbf{T}(k)$ is defined as

$$\mathbf{T}(k) = \lambda\mathbf{G}(k-1) + \mathbf{y}(k)\,\mathbf{e}^H(k). \tag{20}$$

The search direction vectors for the next update can be computed as

$$\mathbf{U}(k+1) = \mathbf{G}(k) + \mathbf{U}(k)\,\beta(k), \tag{21}$$

where the ith diagonal element of the matrix $\beta(k)$ can be computed as

$$\beta_i(k) = \frac{\left(\mathbf{g}_i(k) - \mathbf{g}_i(k-1)\right)^H \mathbf{g}_i(k)}{\mathbf{g}_i^H(k-1)\,\mathbf{g}_i(k-1)}, \tag{22}$$

where $\mathbf{g}_i(k)$ is the ith column of the gradient matrix $\mathbf{G}(k)$. Now all the linear systems are constantly updated by the modified conjugate gradient algorithm. This is computationally complex. Using the Galerkin projections, an approximate solution can be obtained by updating through just one seed system j at each instant, while the other systems are updated through the projections, i.e., they use the search direction of the seed system as

$$\mathbf{w}_i(k) = \mathbf{w}_i(k-1) + \mathbf{u}_j(k)\,\alpha_i(k), \tag{23}$$

where the step size $\alpha_i(k)$ is selected as

$$\alpha_i(k) = \frac{\mathbf{u}_j^H(k)\,\mathbf{t}_i(k)}{\mathbf{u}_j^H(k)\,\Phi(k)\,\mathbf{u}_i(k)}, \quad i = 1, \ldots, M \tag{24}$$

and $\mathbf{u}_j(k)$ is the search direction of the seed system. This scheme although reduces the complexity but at the expense of the small performance degradation.

IV. Performance Review

In this section, we will highlight some of the performance enhancements achieved through the techniques reviewed in the previous section.

In [12], simulations are carried out for the case of coded and uncoded BER performance for MIMO-DFE system with ideal channel estimation and with channel estimation error (20dB). The DFE was tested for 2×2 and 4×4 MIMO system with a QPSK modulation scheme. In case of ideal channel estimation, as the SNR increases, the constrained MIMO-MMSE-DFE equalizer performs much better than the conventional DFE. When the channel estimation error are taken into account, the DFE outperforms the conventional DFE. These performance gains are particularly noticeable in the SNR range of 10dB ¡ SNR ¡ 20dB.

The cyclic MMSE technique in [10], is shown to have significant BER performance enhancement for wide band and narrow band jammers as compared to its linear equivalent. This justifies their use in such scenarios.

Simulation results in [9] show that the proposed limited feedback scheme provides a significantly better performance with a lower feedback rate than the existing schemes in which the detection order is fed back to the transmitter. There is an SNR gain of almost 6dB for the proposed scheme compared to the ordering feedback2 ZF-DFE at a BER = 10^{-3}.

The adaptive channel aided technique in [11] in shown to exhibit better MSE performance than the conventional MIMO-DFE. The SER performance for the static Proakis C channel and time-varying channel against the step size reveals an optimum step size for which the SER is minimized. Also the SER is lower than the conventional DFE for the same step size.

The technique of adaptive conjugate gradient DFE (MIMO-MCG-PI & PII) in [8] exhibits almost identical convergence behaviour compared to the MIMO-RLS algorithm but have been designed for lower computational complexity. It can also be seen that the proposed algorithm has a slightly better MSE performance when tracking a time-varying channel.

V. Simulation

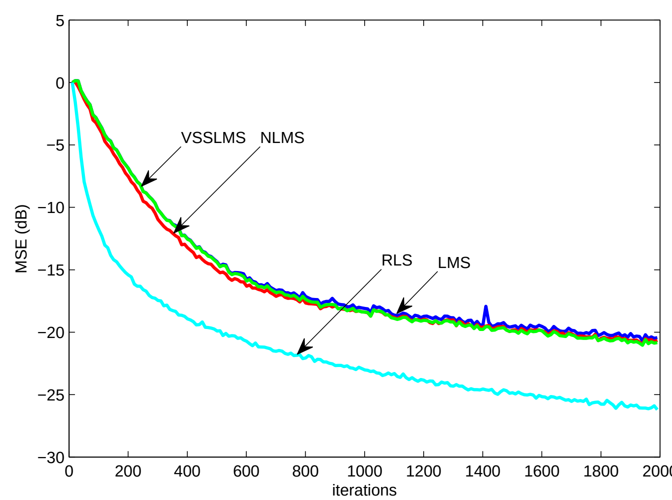

Fig. 7. MSE performance of various adaptive algorithms for a 2×2 MIMO-DFE receiver.

In this section, we have produced mean-square-error (MSE) comparison for different adaptive algorithms in a 2×2 MIMO system employing DFE receiver structure. The algorithms that have been compared are LMS, VSSLMS, NLMS and RLS. All parameters for all the algorithms have been chosen to achieve the maximum rate of convergence. The channel used is the same as used in [8] and the SNR was set to 16dB. Figure (V) shows the MSE performance of the algorithms. It can be seen the RLS algorithm outperforms all the other algorithms in terms of the MSE achieved and convergence rate. Although this performance is at the expense of the complexity of RLS.

VI. Conclusion

In this paper, we have reviewed some of the existing techniques for the MIMO-DFE receivers. Their methodologies were explained and the performance enhancements of each was outlined. It is found from simulations that the RLS algorithm outperforms all the other algorithms in terms of the MSE achieved and the convergence rate at the expense of the computational complexity.

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